Dynamical Basis of the Sum Rule $2\Xi_-^- = \Lambda_- + \sqrt{3}\Sigma_0^+$*

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The amplitudes for the nonleptonic decay of hyperons are explained by a simple dynamical model in the framework of SU(3) symmetry. The parity-conserving amplitudes are described by the Feldman-Matthews-Salam model, while the parity-violating amplitudes are given by the dominance of the $K^*\rightarrow\pi$ diagram. An effective weak Hamiltonian is used that transforms like λ_6 under SU(3). The model predicts the sum rule $2\Xi_-^- = \Lambda_- + \sqrt{3}\Sigma_0^+$ for each set of amplitudes; in addition, all other experimental data on the amplitudes can be fitted by the model. These results are obtained without explicitly invoking *RP* invariance.

I. INTRODUCTION

OCTUPLET transformation properties of nonleptonic weak interactions—the assumption that the nonleptonic weak interaction Lagrangian transforms like a member of an octuplet under SU(3)—have been discussed by a number of authors.[1–7] One of the present authors[2] and Coleman, Glashow, and Lee[8] have shown that the assumptions that: (i) the weak Hamiltonian transforms like[9] λ_6 and (ii) the weak Hamiltonian is approximately *RP* invariant lead to the following sum rules for the covariant decay amplitudes.[10]

S wave:

$$2A(\Xi_-^-) = A(\Lambda_-) + (\tfrac{3}{2})^{1/2} A(\Sigma_-^-)\,. \tag{1}$$

$$A(\Sigma_+^+) = 0\,. \tag{1'}$$

P wave:

$$2B(\Xi_-^-) = B(\Lambda_-) + (\tfrac{3}{2})^{1/2}[B(\Sigma_-^-) - B(\Sigma_+^+)]\,. \tag{2}$$

Gell-Mann,[3] on the other hand, has shown that assumption (i) can be derived from (a) *CP* invariance of weak interactions and (b) current × current nature of weak Langrangian with the currents belonging to an octuplet, and "selective enhancement"[3] of the octuplet channel. In addition, he has shown that (i) alone implies a weaker form of (1) and (1′)[11]:

$$2A(\Xi_-^-) = A(\Lambda_-) + (\tfrac{3}{2})^{1/2}[A(\Sigma_-^-) - A(\Sigma_+^+)]\,. \tag{3}$$

In the present article we wish to present a dynamical model which gives relations (1), (1′), and (2) without resorting to approximate *RP* invariance. In Sec. II, we discuss the parity conserving (p.c.; *P*-wave) amplitudes. The spurion (S_6) which mediates decay processes in the p.c. channel is normal (charge conjugation parity +). We show that the pole dominance model of Feldman, Matthews, and Salam (FMS)[12] is applicable for p.c. amplitudes (since the spurion is normal; see Sec. II) and gives Eq. (2) independently of the choice of parameters (essentially F/D ratios in baryon-baryon-p.s. meson coupling and baryon-baryon-spurion coupling). It is further shown that with a reasonable choice of parameters, all observable p.c. amplitudes are accounted for in this model.

Section III is devoted to the discussion of the parity-violating (p.v.; *S*-wave) amplitudes. Since the p.v. spurion is abnormal (charge conjugation parity −) it cannot be coupled to either a baryon-antibaryon pair or to a pair of p.s. mesons when the members of the pair belong to the same octet. Hence, the FMS model is untenable for this channel if assumption (i) is to hold. Our model for this channel assumes the dominance of the $K^*\rightarrow\pi$ diagram and weak interaction mediating the process $K^*\rightarrow\pi$.[13] It will be shown that if the vector boson octuplet (ρ, K^*, φ) is coupled to the conserved vector currents as in the theory of Sakurai,[14] our model is automatically *RP* invariant. This model predicts $A(\Xi_-^-) = A(\Lambda_-)$ in addition to (1) and (1′), all of which appear to be compatible with experiments. Thus, the present model provides a dynamical rationale for the approximate *RP* invariance of this channel. It is to be emphasized that the baryon poles do not appear in the p.v. amplitudes due to the transformation properties of the p.v. spurion.

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[9] We follow the notation of M. Gell-Mann, Phys. Rev. **125**, 1067 (1962); and (unpublished).
[10] The symbols used in Eqs. (1), (1′), and (2) are defined in Ref. 2.
[11] Incidentally, Eqs. (2) and (3) are in excellent accord with experiment: see D. Duane Carmony *et al.*, Phys. Rev. Letters **12**, 482 (1964) and (unpublished); M. L. Stevenson *et al.*, Phys. Letters **9**, 349 (1964).
[12] G. Feldman, P. T. Matthews, and A. Salam, Phys. Rev. **121**, 302 (1961). H. Sugawara, Nuovo Cimento **31**, 635 (1964) discusses the same model for both the p.c. and p.v. amplitude. Y. Hara, Phys. Rev. Letters **12**, 378 (1964) discusses a related model, but unfortunately his Table I contains a serious error [The contribution from $B_8 = B_{8,0}^*$ in Hara's Table I is in disagreement with Sugawara's Eq. (6); we have independently verified Sugawara's equations.]
[13] This model has also been considered by J. Schwinger, Phys. Rev. Letters **12**, 630 (1964). We thank Professor K. T. Mahanthappa for calling our attention to this paper.
[14] J. J. Sakurai, Ann. Phys. (N. Y.) **11**, 1 (1960). In a model such as Schwinger's [J. Schwinger, Phys. Rev. **135**, B816 (1964)], this is not the case.

II. THE p.c. AMPLITUDE

The pole-dominance model of Feldman, Matthews, and Salam has previously been applied to the nonleptonic decays of hyperons by Sugawara.[12] He applied the model to both the p.c. and the p.v. amplitudes and obtained results that agreed only fairly well with experiment. As mentioned in the Introduction, and shown below, the model is applicable only to the p.c. amplitude. The FMS model assumes that the three diagrams in Fig. 1 dominate the decay processes. The three-particle vertex is described by strong interactions in an SU(3) invariant fashion; the two-particle vertex may be described by an effective weak Hamiltonian of the form

$$H_W = f\,\mathrm{Tr}\bar{B}[B,\lambda_6] + d\,\mathrm{Tr}\bar{B}\{B,\lambda_6\} + g\,\mathrm{Tr}\lambda_6 MM\,, \quad (4)$$

where the fact that H_W transforms like λ_6 has been explicitly displayed. B and M are the 3×3 matrix representations of the baryon and meson octuplets. Equation (4) is CP invariant, as well as parity conserving. Parity violating terms in (4) would take the form of a γ_5 inserted in the baryon part.

$$H_W^{\mathrm{p.v.}} = f'\,\mathrm{Tr}\bar{B}\gamma_5[B,\lambda_6] + d'\,\mathrm{Tr}\bar{B}\gamma_5\{B,\lambda_6\}\,. \quad (5)$$

Under the CP operation Eq. (5) changes sign, verifying our assertion that the FMS model cannot describe the p.v. amplitudes.

Equation (4), coupled with the usual SU(3) invariant description of the baryon-baryon-pseudoscalar meson vertex, leads directly to Sugawara's results[12] for the P-wave amplitudes. We reproduce them here with the addition of the meson term:

$$B(\Lambda_-) = \frac{g}{\sqrt{6}}\left[-\frac{(D+F)}{M_\Sigma - M_N}(1+\gamma) + \frac{(3F-D)}{M_\Lambda - M_N} - \frac{C(2-\gamma)}{m_\pi^2 - m_K^2}\right],$$

$$B(\Sigma_+^+) = \frac{g(1+\gamma)}{6}\left[\frac{3(F+D)}{M_\Sigma - M_N} - \frac{(3F-D)}{M_\Lambda - M_N}\right]$$

$$B(\Sigma_-^-) = \frac{g}{6}\left[\frac{(-3F+D)}{M_\Lambda - M_N}(1+\gamma) + \frac{(3F+3D)}{M_\Sigma - M_N}(1-\gamma) + \frac{6\gamma C}{m_\pi^2 - m_K^2}\right], \quad (6)$$

$$B(\Xi_-^-) = \frac{g}{\sqrt{6}}\left[-\frac{(3F+D)}{M_\Xi - M_\Lambda}\gamma + \frac{(F-D)}{M_\Xi - M_\Sigma}(1+\gamma) - \frac{C(1-2\gamma)}{m_\pi^2 - m_K^2}\right],$$

where the SU(3) symmetric baryon-baryon-p.s. meson coupling is written as[15]

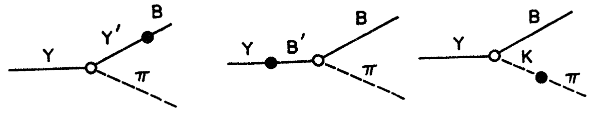

FIG. 1. The pole diagrams contributing to the p.c. decay amplitude in the FMS model.

$$g[\mathrm{tr}\bar{B}\gamma_5 MB + \gamma\,\mathrm{tr}\bar{B}\gamma_5 BM].$$

F and D are the invariant amplitudes describing the baryon weak vertex, while C is the amplitude for the meson vertex. Equation (6) may be simplified considerably by making the approximations $M_\Lambda \simeq M_\Sigma$ and $M_\Xi - M_\Sigma = M_\Sigma - M_N = \Delta M$. Inasmuch as the experimental data to be fitted have large uncertainties, this is a reasonable approximation. We have then

$$\begin{aligned} B(\Lambda_-) &= (F'/\sqrt{6})(2-\gamma) - (D^1/\sqrt{6})(2+\gamma)\,,\\ B(\Sigma_+^+) &= \tfrac{2}{3}D'(1+\gamma)\,,\\ B(\Sigma_-^-) &= -\gamma F' + D'(2-\gamma)/3\,,\\ B(\Xi_-^-) &= [(1-2\gamma)/\sqrt{6}]F' - (D'/\sqrt{6})(1+2\gamma)\,, \end{aligned} \quad (7)$$

where

$$D' = gD/\Delta M\,, \quad F' = gF/\Delta M - gC/(m_\pi^2 - m_K^2)\,.$$

In this approximation the meson vertex does not make an independent contribution. In Eq. (7) there are three parameters to fit four amplitudes. The sum rule, Eq. (2), is satisfied without any restriction on the parameters. The ratio γ is predicted from strong interactions to be on the order of $\frac{1}{2}$.[15] Using $\gamma = 0.29$, we are able to fit the experimental values of $B(\Lambda_-)$ and $B(\Xi_-^-)$ together with $B(\Sigma_-^-) = 0$. Specifically, if $B(\Lambda_-) = 2.02\times10^5\ \mathrm{sec}^{-1} m_\pi^{-1}$ and $B(\Xi_-^-) = -1.41\times10^5\ \mathrm{sec}^{-1} m_\pi^{-1}$,[11] we find[16]

$$\gamma = 0.29\,, \quad F'/D' = 1.96\,, \quad D' = 4.6\times10^5\ \mathrm{sec}^{-1} m_\pi^{-1}\,. \quad (8)$$

These values seem to be physically reasonable.

III. THE p.v. AMPLITUDES

In Sec. II it was shown that the FMS pole dominance model cannot be used to explain the p.v. amplitudes. However, there is another simple model that does describe these amplitudes in a consistent fashion. We

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[15] The parameter γ is related to de Swart's α [J. J. de Swart, Rev. Mod. Phys. **35**, 916 (1963)] by $\gamma = -2\alpha + 1$. Our γ is identical with Sugawara's (Ref. 12) α. $\gamma \simeq \frac{1}{2}$ corresponds to $\alpha \simeq \frac{1}{4}$, the value suggested by de Swart.

[16] If, instead of using Eq. (7), we use Eq. (6) to fit the experimental data, we find four independent parameters. Equation (2), together with the values of $B(\Sigma_-^-)$, $B(\Lambda^-)$, $B(\Xi_-^-)$ given above Eq. (8), gives the following values for the parameters:

$$\gamma = 0.28\,, \quad F/D = 2.43\,, \quad [gC/(m_\eta^2 - m_K^2)](\Delta M_{\mathrm{av}} = 1.87\,, \quad gD/\Delta M_{\mathrm{av}} = 19.0\times10^5\ \mathrm{sec}^{-1} m_\eta^{-1}\,,$$

where $\Delta M_{\mathrm{av}} = \frac{1}{2}(M\ \ - M_N)$. Note that the value of γ is virtually unchanged.

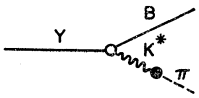

FIG. 2. The $K^* \to \pi$ diagram which provides the dominant contribution to the p.v. amplitudes.

assume the dominance of the K^* pole with weak interactions mediating the $K^* \to \pi$ process.[17] The diagram for this process is given in Fig. 2. The baryon-baryon-vector meson vertex is governed by strong interactions in an SU(3) invariant fashion. Moreover, the vector mesons are assumed to be coupled to a conserved baryon current so that only f-type coupling enters; the conserved current hypothesis implies, in addition, that two form factors are sufficient to describe the space time properties of the vertex. The weak vertex is described by a parity violating, CP invariant, Hamiltonians of the form

$$H_W = \alpha \operatorname{tr}\lambda_6\{V_\mu, \partial_\mu M\}, \tag{9}$$

where V_μ is the vector meson matrix. This Hamiltonian has $P=-1$; moreover, it has $R=-1$. Thus, it is RP invariant and relations (1) and (1′) will hold.

The matrix element corresponding to Fig. 2 has the form[18]

$$\begin{aligned} \bar{u}_B A(YB) u_Y &= \bar{u}_B(a_{YB}\gamma_\mu + c_{YB}\sigma_{\mu\nu}q_\nu)u_Y \\ &\quad\times\left(\delta_{\mu\lambda} - \frac{q_\mu q_\lambda}{m_{K^*}{}^2}\right)(m_\pi{}^2 - m_{K^*}{}^2)^{-1}\alpha_{K^*\pi}q_\lambda \qquad (10)\\ &= \frac{M_Y - M_B}{m_{K^*}{}^2} a_{YB}\alpha_{K^*\pi}\bar{u}_B u_Y , \end{aligned}$$

where $q = p_Y - p_B = p_\pi$. The $K^{*-} \to \pi$ vertex is described by $\alpha_{K^*\pi}$, while the strong vertex is given by a_{YB}. If $M_Y - M_B = \Delta M$ is assumed constant, all the factors in (10), except for a_{YB}, are independent of the particular decay mode under consideration. Thus, we have

$$A(YB) = a_{YB}{}' \equiv (\Delta M / m_K{}^2)\alpha_{K^{*-}\pi} a_{YB}, \tag{11}$$

where the various $a_{YB}{}'$ are given in terms of a single parameter by the assumption of SU(3) invariant f-type coupling of the baryons to the K^{*-}. $A(\Sigma_+{}^+)$ is automatically zero since coupling to K^{*+} would not conserve strangeness at the strong vertex. The parity violating amplitudes are given by[19]

$$\begin{aligned} A(\Lambda_-) &= (\tfrac{3}{2})^{1/2} f, \\ A(\Sigma_+{}^+) &= 0, \\ A(\Sigma_-{}^-) &= f, \\ A(\Xi_-{}^-) &= (\tfrac{3}{2})^{1/2} f. \end{aligned} \tag{12}$$

Relations (1) and (1′) are indeed satisfied. There is the additional prediction that $A(\Lambda_-) = A(\Xi_-{}^-)$. Experimentally it is known that $A(\Lambda_-) = 3.1 \times 10^5$ sec$^{-1}m_\pi{}^{-1}$ and $A(\Xi_-{}^-) = 4.1 \times 10^5$ sec$^{-1}m_\pi{}^{-1}$;[11] thus in the light of the crudeness of the model, the agreement between theory and experiment is satisfactory.[20]

Equation (10) vanishes in the limit of exact SU(3) symmetry; however, it is the dominant term when the symmetry is broken. We have estimated the other effects of symmetry breaking interactions.[21] One is due to the induced scalar coupling between baryons and vector mesons. This effect is estimated on the basis of the model due to Nambu and Sakurai[22] and is found to be of order of $(m_\pi/m_\kappa)^2$ compared to the main effect, where m_κ is the mass of a scalar meson of strangeness ± 1 and isotopic spin $\frac{1}{2}$ [we hesitate to identify this with the 725-MeV resonance. It is, however, reasonable to say that $m_\kappa \geq m_K$]. The second is the induction of the D-type coupling. A convincing estimate of this effect appears impossible. We merely note that the sum rule, *Eq. (1), and Eq. (1′) are still valid independent of the D-type admixture in the current.* In fact some D-type admixture will allow the model to fit the experiments better.

[17] On the mass shell, the process $K^* \to \pi$ is forbidden by the conservation of angular momentum. However, when K^* is virtual, the Feynman diagram Fig. 2 generates a contact term in the S wave. It means, in dispersion theory, that the K^* intermediate state does not contribute to the absorptive part, but rather to the subtraction constant.

[18] We use the notation of H. Sugawara (Ref. 12).

[19] If one does not assume that K^* is coupled to a conserved current, Eqs. (1) and (1′) still hold. However, the relation $A(\Lambda_-) = A(\Xi_-{}^-)$ does not.

[20] Even if the mass differences are not all taken to be equal, $A(\Sigma_+{}^+)$ is still predicted to be identically zero in this model. Equation (12) should have each term multiplied by $M_Y - M_B / \Delta M_{\text{av}}$; the sum rule for the p.v. amplitude is then satisfied to ten percent and $A(\Xi_-{}^-) = 1.16\, A(\Lambda_-)$.

[21] We thank Professor S. Coleman for an interesting discussion on this point.

[22] Y. Nambu and J. J. Sakurai, Phys. Rev. Letters **11**, 42 (1963).